The value of the first derivative (slope) of the function

$$E(t) = \int_0^T I(t)V(t)dt = \int_0^T A\sin(\omega t+\Theta)(F+V_m\sin(\omega t))dt \quad (1)$$

at a given point τ within the interval $[0,T]$ is

$$E'(\tau) = \frac{d}{d\tau}\int_0^\tau I_{in}V_{in}dt = I_{in}V_{in}|_{t=\tau} = A\sin(\omega\tau+\Theta)(F+V_m\sin(\omega\tau)) \quad (2)$$

where V_m is the amplitude of the applied voltage, V; F is the offset voltage, V; $\omega = 2\pi f$ is the angular velocity, rad s^{-1}; $f = \frac{1}{T}$ is the frequency, Hz; T is the period, s, t is the time, s, $A = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{2\pi fC}\right)^2}}$ and $\Theta = arctan\left(\frac{1}{R2\pi fC}\right)$.

Therefore, the average value of the first derivative (slope) of that function within the entire interval $[0,T]$ is

$$P_{in}^{from\ integral} = \frac{1}{T}\int_0^T E'(t)dt = \frac{1}{T}\int_0^T A\sin(\omega t+\Theta)(F+V_m\sin(\omega t))dt = \frac{AVmcos(\Theta)}{2} = const \quad (3)$$

On the other hand, since, as seen, $A\sin(\omega\tau+\Theta)(F+V_m\sin(\omega\tau))$ is the slope of the function $E(t)$ at time τ the average slope within the interval $[0,T]$ can be expressed as:

$$P_{in}^{from\ series} = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n} Asin\left(\omega\tau+\Theta\right)\left(F+V_m sin\left(\omega\tau\right)\right)\right) \quad (4)$$

where $\tau = \left(\frac{T}{n}+\frac{(i-1)T}{n-1}\right)$. As is known the value of the integral (eq.(3)) is the limit of the corresponding Riemann sum while series (eq.(4)) is not a Riemann sum although expressing the same thing as (eq.(3)). Nevertheless, it is expected that eq.(3) and eq.(4) should produce the same result.

In this exercise we would like to check this out.

Since there is no analytical way to check what the series (eq.(4)) converges to one way to find that out is to use a numerical method. Numerical methods are based inherently on partitioning the studied interval $[0,T]$. Too small a partition would lead to approximation while too big a partition will lead to greater rounding errors as well as floating point errors. Therefore, for this exercise, a partition $P = 1000$ is chosen as a compromise.

It can be demonstrated that when numerical integration is carried out for $P = 1000$ both for offset $F \neq 0$ and for offset $F = 0$ the result is practically constant for all values of F and is practically equal to the value $\frac{AVmcos(\Theta)}{2} = const$ of the integral in eq.(3).

The numerical calculation of the series in eq.(4) for the same $P = 1000$ and offset $F = 0$ also gives as a result a value practically equal to $\frac{AVmcos(\Theta)}{2} = const$.

However, when the numerical calculation of the series in eq.(4) is carried out for the same $P = 1000$ but the offset now is $F \neq 0$ then the result becomes a function of the offset F. For values of $F < 0$ not only the integral tends towards zero but after a certain F it becomes negative. The opposite is observed when $F > 0$. In this case the integral becomes more and more positive with the increase of F. Of course, there are physical limits to the decrease (respectively increase) of F. However, the observed dramatic effects in changing the $P_{in}^{from\ series}$ value compared to the constant value of the $P_{in}^{from\ integral}$ is observed even at modest physically viable values of V_m and F on the order of volts.