NANAColt Engineering, LLC

A

Calculations Chart

Subject		Sheet Number of
File	By	Date

Joule-Thompson throttling - starting with Maxwells equations

$$ds = \left(\frac{\partial s}{\partial P}\right)_T dP + \left(\frac{\partial s}{\partial T}\right)_P dT \qquad (1)$$

$$dh = T\,ds + v\,dP \quad \text{(substitute ds into this equation)} \qquad (2)$$

$$dh = T\left[\left(\frac{\partial s}{\partial P}\right)_T dP + \left(\frac{\partial s}{\partial T}\right)_P dT\right] + V\,dP \qquad (3)$$

$$dh = T\left(\frac{\partial s}{\partial T}\right)_P dT + \left[T\left(\frac{\partial s}{\partial P}\right)_T + V\right] dP \qquad (4)$$

Maxwell Relation

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \quad \text{substitute this relationship into equation (4)}$$

$$dh = T\left(\frac{\partial s}{\partial T}\right)_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] dP$$

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_P \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad \text{(thermal expansion coefficient for liquids)}$$

$$\boxed{dT = \frac{1}{C_p}\,dh + \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] dP}$$

$$T = T(h, P)$$

$$dT = \left(\frac{\partial T}{\partial h}\right)_P dh + \left(\frac{\partial T}{\partial P}\right)_h dP$$

$$\frac{1}{C_p} = \left(\frac{\partial T}{\partial h}\right)_P \quad , \quad \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] = \left(\frac{\partial T}{\partial P}\right)_h = \mu_J$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{c_p/c_v \cdot C_p}{K_s \cdot V}$$

$$\mu_J = \left(\frac{V}{C_p}\right)(T\beta - 1) \rightarrow \boxed{\Delta T = \left(\frac{V}{C_p}\right)\left(T \cdot \frac{1/c_v}{K_s \cdot V} - 1\right) \cdot \Delta P}$$

Redlich / Kwong equation of state

$$Z = \frac{1}{1-h} - \frac{a}{bRT^{1.5}}\left(\frac{h}{1+h}\right), \quad h = \frac{bP}{ZRT} = \frac{b}{V}$$

$$T_r = \frac{T}{T_c}, \quad P_r = \frac{P}{P_c} \qquad P_c = 651.9 \text{ psig} \quad T_c = -180.86\,°F$$

(Methane)

$$a = \frac{0.42748\,R^2 T_c^{2.5}}{P_c} \qquad b = \frac{0.08664\,R\,T_c}{P_c}$$

$$Z = \frac{1}{1-h} - \frac{4.9340}{T_r^{1.5}}\left(\frac{h}{1+h}\right) \qquad h = \frac{0.08664\,P_r}{Z\,T_r}$$

$$\Delta T = \left(\frac{ZRT_1}{P_1} \cdot \frac{1}{C_p}\right)\left(T_1 \cdot \frac{1/c_v}{K_s} \cdot \frac{P_1}{ZRT_1} - 1\right) \cdot \Delta P$$

$$\boxed{T_2 = \left(ZRT_1 \cdot \frac{1}{C_p}\right)\left(T_1 \cdot \frac{1/c_v}{K_s} \cdot \frac{P_1}{ZRT_1} - 1\right) \cdot (P_2 - P_1) + T_1}$$

For methane:

$$C_p = A + \frac{B}{2} \cdot T_0(\tau + 1) + \frac{C}{3} \cdot T_0^2(\tau^2 + \tau + 1) + \frac{D}{\tau \cdot T_0^2} \qquad \tau = \frac{T}{T_0}$$

A =

B =

C =

D =

C_p =

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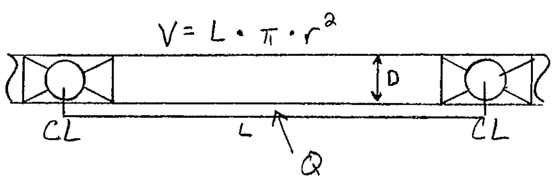
$$\frac{(T_1 - T_2)}{(P_1 - P_2)} = \left(\frac{V \cdot T_1 \cdot \beta}{Cp \cdot m} - \frac{V}{Cp \cdot m}\right)$$ (first order non-seperable differential equation or iterate)

Unit Check

$$°F = \left(\frac{ft^3 \cdot °F \cdot 1/°F}{\frac{ft-lbf}{lbm-°F} \cdot lbm} - \frac{ft^3}{\frac{ft-lbf}{lbm-°F} \cdot lbm}\right) \cdot \frac{lbf}{ft^2} \rightarrow °F = °F$$

Thermal Expansion of Heat Trace Failure - Closed system



Fluid = Methanol

$T_1 = 70°F$	$V =$
$T_2 = 150°F$	$\beta =$
$P_1 = 14.7$ psia	$Cp =$
$P_2 =$ unknown	$M =$

Solution of first order differential equation

$$\left(\frac{T_1 - T_2}{P_1 - P_2}\right) = \left[\frac{V \cdot T_1 \cdot \beta}{Cp \cdot m} - \frac{V}{Cp \cdot m}\right]$$ where $C_1 = \frac{V \cdot \beta}{Cp \cdot m}$ & $C_2 = \frac{V}{Cp \cdot m}$

$$\frac{dT}{dP} + -C_1 T = -C_2$$

Standard solution to first order non-seperable

$$T_2 = \frac{\int_{P_1}^{P_2} e^h \cdot -C_2 \, dP + T_1}{e^h}$$ where $h = \int_{P_1}^{P_2} -C_1 \, dP = -C_1 P \rightarrow$

$$-C_2 \int_{P_1}^{P_2} e^{-C_1 P} \, dP = -C_2 \cdot \frac{1}{-C_1} e^{-C_1 \Delta P}$$

$$T_2 = \frac{-C_2 \cdot \frac{1}{-C_1} \cdot e^{-C_1 \Delta P} + T_1}{e^{-C_1 \Delta P}} \rightarrow T_2 = \frac{C_2}{C_1} + \frac{T_1}{e^{-C_1 \Delta P}}$$

$$\left(T_2 - \frac{C_2}{C_1}\right) = \frac{T_1}{e^{-C_1 \Delta P}} \rightarrow e^{-C_1 \Delta P} = \frac{T_1}{\left(T_2 - \frac{C_2}{C_1}\right)} \rightarrow \ln e^{-C_1 \Delta P} = \ln\left[\frac{T_1}{\left(T_2 - \frac{C_2}{C_1}\right)}\right]$$

$$-C_1 (P_2 - P_1) = \ln\left[\frac{T_1}{\left(T_2 - \frac{C_2}{C_1}\right)}\right] \rightarrow \boxed{P_2 = \frac{\ln\left[\frac{T_1}{\left(T_2 - \frac{C_2}{C_1}\right)}\right]}{-C_1} + P_1}$$

Second Unit Check

$$\frac{lbf}{ft^2} = \frac{\frac{°F}{\left(°F - \frac{C_2}{C_1}\right)}}{\frac{ft^2}{lbf}} + \frac{lbf}{ft^2}$$

$$\downarrow$$

$$\frac{lbf}{ft^2} = \frac{lbf}{ft^2} + \frac{lbf}{ft^2}$$

$$C_2 = \frac{ft^3}{\frac{ft \cdot lbf}{lbm \cdot °F} \cdot lbm} \rightarrow \frac{ft^3 \cdot °F}{ft \cdot lbf}$$

$$C_1 = \frac{ft^3 \cdot °F^{-1}}{\frac{lbf \cdot ft}{lbm \cdot °F} \cdot lbm} \rightarrow \frac{ft^3}{lbf \cdot ft} \quad ft^2$$

$$\frac{C_2}{C_1} = \frac{ft^3 \cdot °F}{ft \cdot lbf} \cdot \frac{lbf \cdot ft}{ft^3} = °F$$

See attached Math CAD file for the application of the derived equation

By: Ron Pearson

Volume of trapped liquid (ft^3)

$V := 30$

Initial Pressure (lbf/in^2)

$P_1 := 14.7$

Heat capacity of fluid (ft-lbf/lbm-F)

$C_p := 500$

Final Temperature of heat source (F)

$T_2 := 200$

Initial Temperature (F)

$T_1 := 70$

Thermal expanson coefficient (F^-1)

$\beta := .000642$

fluid density (lbm/ft^3)

$\rho := 59.25$

Mass of trapped fluid (lbm)

$m := V \cdot \rho \qquad m = 1777.5$

Initial Pressure (lbf/ft^2)

$P := 2116.8$

Expanded Volume (ft^3)

$V_{new} := V \cdot \left[1 + \beta \cdot \left(T_2 - T_1\right)\right]$

$V_{new} = 32.504$

$$C_1 := \frac{V_{new} \cdot \beta}{C_p \cdot m} \qquad C_2 := \frac{V_{new}}{C_p \cdot m} \qquad \frac{C_2}{C_1} = 1557.632$$

$C_1 = 0.00000002 \qquad C_2 = 0.000037$

Pressure Equation derived from attached calculation (lbf/ft^2)

$$\ln\left(\left|\frac{T_1}{T_2 - \frac{C_2}{C_1}}\right|\right) = -2.965$$

$$P_2 := \frac{\dfrac{\ln\left(\left|\frac{T_1}{T_2 - \frac{C_2}{C_1}}\right|\right)}{-C_1}}{144} + P_1$$

$P_2 = 876960.993$