Queen Mary
University of London

BSc Examination by course unit.

Wednesday 7th May 2014 2:30pm – 5:00pm

PHY5214/214 Thermal & Kinetic Physics Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM

Examiners:

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T.J.S.Dennis

SECTION A. Answer all questions in Section A.

Question A1

Write down Boltzmann's equation relating the entropy, *S*, of a macroscopic equilibrium state to the microscopic properties of the system. Explain any new symbols used.

[4 marks]

Question A2

Write down the Thermodynamic Identity for a paramagnet and define all of the terms.

[4 marks]

Question A3

What is the internal energy of a gas of *N* rigid diatomic molecules in equilibrium at temperature, *T,* in terms of the temperature? What is the average kinetic energy of one of the molecules in terms of *T*?

[6 marks]

Question A4

A system consisting of a gas contained in a cylinder with a frictionless piston is taken around the closed path a $\rightarrow$ b $\rightarrow$ c as shown in figure 1.

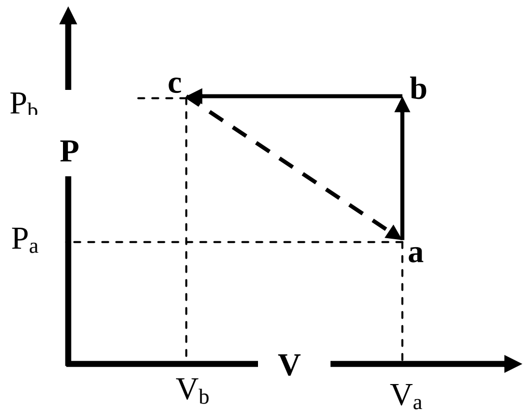

Figure 1

In the isochoric process, a $\rightarrow$ b, 20 J of heat flows ***into*** the gas. What is the change in internal energy in going from a to b? ***Be careful to include the sign.***

[4 marks]

Question A5

i) During the combined process, a $\rightarrow$ b $\rightarrow$ c, shown in figure 1, the net change in the internal energy is $U_c - U_a$ = -150 J and 250 J of heat is expelled by the gas. How much work is done by the gas during this process?

ii) During the direct transition from c $\rightarrow$ a, indicated by the dashed line in figure 1, the gas carries out 80 J of work on the surroundings. What is the heat absorbed during this process?

[6 marks]

Question A6

Write down a definition of the thermal expansion coefficient, β, of a P-V system in terms of thermodynamic variables and a partial differential. Make it clear which thermodynamic variable is being held constant.

[5 marks]

Question A7

A Carnot engine operates between a hot reservoir at temperature, T_1, and a cold reservoir at temperature, T_2, Write down the reversible engine efficiency, η_E , in terms of the reservoir temperatures.

[5 marks]

Question A8

Write down an expression for the Gibbs potential, G, for a P-V-T system.

[5 marks]

Question A9

What is the enthalpy, H, of a gas of N rigid diatomic molecules at equilibrium at temperature T in terms of the temperature and Boltzmann constant?

[5 marks]

Question A10

A heat pump operating between a hot and cold reservoir has an efficiency η_{HP} and accepts heat Q_2 from the cold reservoir. Write down an expression for the heat supplied by this heat pump.

[6 marks]

SECTION B. Attempt two of the four questions in this section.

Question B1

The equation of state of an elastic band under tension, $\mathcal{F}$, at temperature, T, is given by;

$$\mathcal{F} = aT\left[\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2\right].$$

Where L_0 is its free length under zero tension and L is its length under tension while a is a constant.

The Helmholtz free energy is defined as $F = U - TS$ and the incremental work done on an elastic band under tension, $\mathcal{F}$, extended by an infinitesimal length dL is given by $dW = \mathcal{F}dL$.

a)

i) Write down the Thermodynamic Identity for the elastic band.

ii) Using the above definition of F and the Thermodynamic Identity for an elastic band find the natural variables of F.

iii) Find the relationships between the partial differentials of F with respect to each of the two natural variables and two other thermodynamic variables.

iv) From ii) and iii) find a Maxwell relation between partial differentials of S and $\mathcal{F}$ with respect to the natural variables of F.

v) Use this Maxwell relation and the Thermodynamic Identity to show that

$$\left(\frac{\partial U}{\partial L}\right)_T = \mathcal{F} - T\left(\frac{\partial \mathcal{F}}{\partial T}\right)_L.$$

vi) Hence, or otherwise, show that the internal energy U of the elastic band depends only on its temperature and not on its length.

[12 marks]

b) Demonstrate that the thermal expansion coefficient, $\alpha = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_{\mathcal{F}}$, is given by

$$\alpha = -\frac{1}{T}\frac{\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2}{\frac{L}{L_0} + \frac{2L_0^2}{L^2}}.$$

[6 marks]

c) The elastic band is stretched isothermally from L_0 to $2L_0$. What is the heat evolved in this process of stretching? Is this heat absorbed or expelled by the band?

[7 marks]

Question B2

a)

i) State, with an equation, how the entropy of a system is defined up to an additive constant in terms of reversible heat transfer, dQ_R, and the temperature, T, at which the heat transfer occurs. Explain the sign convention.

ii) 1 kg of ice from a freezer at -5^{o}C is placed in a bucket in the garden where the temperature is 20^{o}C. Calculate the entropy change of the ice/water, ***including sign***, after all the ice has melted and the water has come to thermal equilibrium.

iii) Calculate the entropy change of the garden (due to the processes occurring with the ice/water) ***including sign***.

[10 marks]

b)

i) By manipulating the thermodynamic identity for a P-V-T system and the equation of state show that for one mole of an ideal rigid diatomic gas the entropy change, as volume and temperature are changed, is given by;

$$\Delta S = S(V_f, T_f) - S(V_i, T_i) = \frac{5}{2} R \ln\left[\frac{T_f}{T_i}\right] + R \ln\left[\frac{V_f}{V_i}\right].$$

ii) Using the results of **b** i) above demonstrate that for an ideal gas undergoing an adiabatic expansion from initial volume V_i to final volume V_f there is zero change in entropy.

[7 marks]

c) A mass of water, m, at temperature T_1 is mixed with an equal mass of water at temperature T_2 under isobaric and adiabatic conditions.

i) Demonstrate that the entropy change of the universe is given by;

$$\Delta S_{Uni} = 2mc_P \ln\left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right].$$

ii) By considering the above expression for the entropy change of the universe express the requirement that the Second Law is obeyed in terms of T_1 and T_2.

iii) Demonstrate that the Second Law has been obeyed.

[8 marks]

Hint: You may use the fact that $(a-b)^2 \geq 0$ ***for the case when a and b are both real numbers***

Question B3

a)

i) In an isothermal process one mole of a ***rigid diatomic*** ideal gas at temperature, T, is expanded from an initial volume, V_i, to a final volume, $3V_i$. What is the work done in this process in terms of T and V_i ? ***Be careful to include the sign*** and state whether the work was done by or on the gas.

ii) In an isobaric process one mole of a non-***rigid diatomic*** ideal gas at pressure, P, is compressed from initial volume, V_i, to a final volume, $\frac{V_i}{2}$. Write down the work done during this process in terms of P and V_i. ***Be careful to include the sign*** and state whether the work was done by or on the gas.

iii) In an isochoric process one mole of a ***monatomic*** ideal gas of volume, V, has its pressure decreased from initial pressure, P_i, to a final pressure, $\frac{P_i}{2}$. What is the heat transfer during this process in terms of P_i and V? ***Be careful to include the sign*** and state whether the heat was absorbed or released by the gas.

[9 marks]

b)

i) Using the First Law, or otherwise, show that during an adiabatic change a monatomic ideal gas will obey a relation of the form $PV^{\gamma} = constant$ and derive the value of the constant γ. State how γ is related to the heat capacities for the gas.

ii) Find an equivalent rule relating P and T in an adiabatic process.

[7 marks]

c) An engine is constructed with one mole of an ideal diatomic gas as the working substance for which the internal energy is $U = \frac{5}{2}PV$ and the adiabatic constant γ has the value $\gamma = 7/5$. It operates reversibly in a cycle **a**→**b** → **c**→**a**. The process **a**→**b** is an *isochoric* process at volume V_a during which the pressure increases from P_a to P_b and heat Q_1 is absorbed. The process **b**→ **c** is an *adiabatic* expansion back to the initial pressure P_a and volume V_c. The process **c**→**a** is an *isobaric* compression during which heat Q_2 is ejected and the working substance returns to initial state **a**.

i) Sketch the engine cycle in a P-V diagram.

ii) By applying the First Law to the processes **a** → **b** and **c** → **a**, obtain expressions for the heat flows Q_1 and Q_2 in terms of pressures P_a and P_b and the volumes V_a and V_c, corresponding respectively to the states **a**, **b**, **c**.

iii) Show that the efficiency of this engine can be expressed solely in terms of the pressure ratio $r = P_b/P_a$ as

$$\eta_E = 1 - \frac{7}{5}\frac{r^{5/7} - 1}{r - 1}.$$

[9 marks]

Question B4

a) Cavity radiation can be regarded as a photon gas, a P-V-T system whose internal energy U is related to its pressure and volume by $U = 3PV$. Moreover the energy density, u, defined by $u = U/V$, depends only on the temperature of the gas, T, as

$$u(T) = \frac{4\sigma}{\mathrm{c}} T^4$$

where σ and c are universal constants given in the list of constants.

i) From the information above obtain the equation of state for the cavity radiation by expressing P in terms of T.

ii) Calculate the pressure of a photon gas at a temperature of 1.00×10^6 K.

iii) Calculate the temperature of a photon gas whose pressure is 1.00 atm.

[6 marks]

b) A photon gas undergoes a reversible adiabatic compression from an initial state T_1, V_1 to a final state T_2, V_2. The entropy of the gas can be expressed as

$$S = \frac{16\sigma}{3c} T^3 V .$$

i) How does the temperature vary with volume during this process?

ii) How does the pressure vary with volume during this process?

iii) Show that the work done on the gas during the compression is proportional to the change in temperature $T_2 - T_1$.

[8 marks]

c) A deep space probe consists of a spherical module with a blackened surface of area A_0 and a radius R = 1.00 m. Inside the module is a nuclear power source which can supply heat at a rate $\dot{Q} = 5.00$ kW.

i) Describe the energy balance for the module and show that the steady state surface temperature T_0 of the module is 289 K.

ii) The probe enters a region of space filled with electromagnetic radiation with a flux $\phi = 1.4 \times 10^2$ Wm^{-2}. Describe the new energy balance and show that the steady state surface temperature T_1 is now given by

$$T_1 = \left(\frac{\dot{Q}}{\sigma A_0} + \frac{\phi}{\sigma} \right)^{1/4}$$

iii) Calculate the numerical value of T_1.

iv) The probe is equipped with a blackened panel which it can deploy externally to increase its total surface area from A_0 to A_1. What value of A_1 will bring the steady state surface temperature back to T_0? **[11 marks]**

End of Paper - An appendix of 1 page follows

DATA SHEET

You may wish to use some of the following data.

k_B	= Boltzmann's constant	=	1.38×10^{-23} J K^{-1}
σ	= Stefan's constant	=	5.67×10^{-8} J s^{-1} m^{-2} .K^{-4}
c	= Velocity of light in vacuo	=	3.0×10^{8} m s^{-1}
N_A	= Avagadro's number	=	6.02×10^{23} mol^{-1}
R	= Gas constant	=	8.31 J mol^{-1} K^{-1}
P_{atm}	= Atmospheric pressure	= 1atm =	1.01×10^{5} Pa
T_S	= Ice point of water	=	273.15 K
c_P	= Specific heat of water at constant pressure	=	4.2×10^{3} J K^{-1} kg^{-1}
c_P^{Ice}	= Specific heat of ice at constant pressure	=	2.1×10^{3} J kg^{-1}
l^{SL}	= Latent heat of melting ice	=	3.33×10^{5} J kg^{-1}
1 *amu*	= One atomic mass unit	=	1.66×10^{-27} kg