

How many modes per unit wavelength?

Having developed an expression for the [number of standing wave modes in a cavity](#), we would like to know the distribution with wavelength. This may be obtained by taking the derivative of the number of modes with respect to wavelength.

$$\frac{dN}{d\lambda} = \frac{d}{d\lambda} \left[\frac{8\pi L^3}{3\lambda^3} \right] = -\frac{8\pi L^3}{\lambda^4}$$

The negative sign here reveals that the number of modes decreases with increasing wavelength. Now to get the number of modes per unit volume per unit wavelength, we can simply divide by the volume of the cubical cavity.

$$\frac{\text{Number of modes per unit wavelength}}{\text{Cavity volume}} = -\frac{1}{L^3} \frac{dN}{d\lambda} = \frac{8\pi}{\lambda^4}$$

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Why is there a minus sign here?

$dN/d\lambda$ is already having a minus sign from the earlier step,

And in this step we are just supposed to divide by L^3 .

I am unable to see the reason for this minus sign.

How much energy per unit volume?

Assigning energy to the electromagnetic standing waves in a cavity draws on the principle of [equipartition of energy](#). Each standing wave mode will have average energy kT where k is Boltzmann's constant and T the temperature in Kelvins. Letting u represent the energy density

$$\frac{du}{d\lambda} = \frac{1}{L^3} \frac{dE}{d\lambda} = -kT \frac{1}{L^3} \frac{dN}{d\lambda} = \frac{8\pi kT}{\lambda^4}$$

This is an important relationship in classical electromagnetic cavity theory. It can also be expressed in terms of the frequency ν by making use of the chain rule and the wave relationship:

$$\frac{du}{d\nu} = \frac{du}{d\lambda} \frac{d\lambda}{d\nu} = \frac{du}{d\lambda} \frac{d(c/\nu)}{d\nu} = \frac{du}{d\lambda} \frac{-c}{\nu^2}$$

Why is there a minus sign here?

$dN/d\lambda$ is already having a minus sign from the earlier step,

And in this step we are just supposed to divide by L^3 and multiply by kT .

I am unable to see the reason for this minus sign.

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