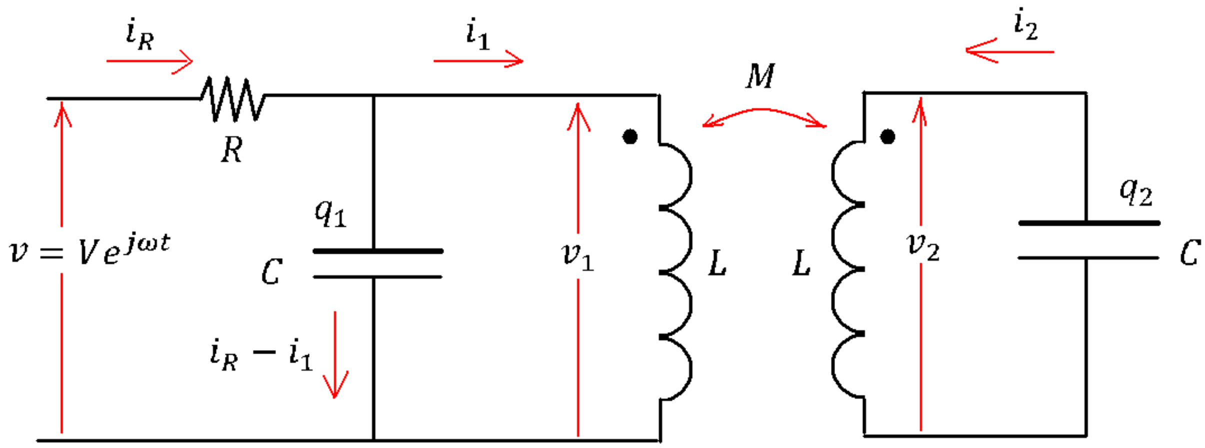
a. **<u>Case 1</u>: Connected in parallel:**



I suppose that the resistance in the branch L//C isn't large, and the internal resistance of the voltage source R is significant. Besides, to simplify the calculation, I choose capacitors of the same capacitance C and inductors of the same inductance L, as I expect that resonance of the interaction between the 2 circuits occurs when their natural frequencies are the same.

From that, I obtain these equations:

$$v = Ve^{j\omega t} = Ri_R + v_1 \quad \text{..........................(1)}$$

$$v_1 = \frac{q_1}{C} = L\frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{..........................(2)}$$

$$\frac{dq_1}{dt} = i_R - i_1 \quad \text{..(3)}$$

$$v_2 = \frac{q_2}{C} = L\frac{di_2}{dt} + M\frac{di_1}{dt} \quad \text{..........................(4)}$$

$$\frac{dq_2}{dt} = -i_2 \quad \text{..(5)}$$

From (1) and (2): $$Ve^{j\omega t} = Ri_R + L\frac{di_1}{dt} + M\frac{di_2}{dt}$$

From (2) and (3): $$LC\frac{d^2i_1}{dt^2} + MC\frac{d^2i_2}{dt^2} = i_R - i_1$$

From (4) and (5): $$LC\frac{d^2i_2}{dt^2} + MC\frac{d^2i_1}{dt^2} = -i_2$$

Because the source is sine signal, all the currents should be in the sine form. Therefore, from the 3 above resultant equations:

$$Ve^{j\omega t} = Ri_R + j\omega L i_1 + j\omega M i_2$$

$$-\omega^2 LC i_1 - \omega^2 MC i_2 = i_R - i_1$$

$$-\omega^2 LC i_2 - \omega^2 MC i_1 = -i_2$$

From the above simultaneous equations, solve for i_2, with the condition that $\omega = {}^{1}/_{\sqrt{LC}}$ (resonance in the left circuit):

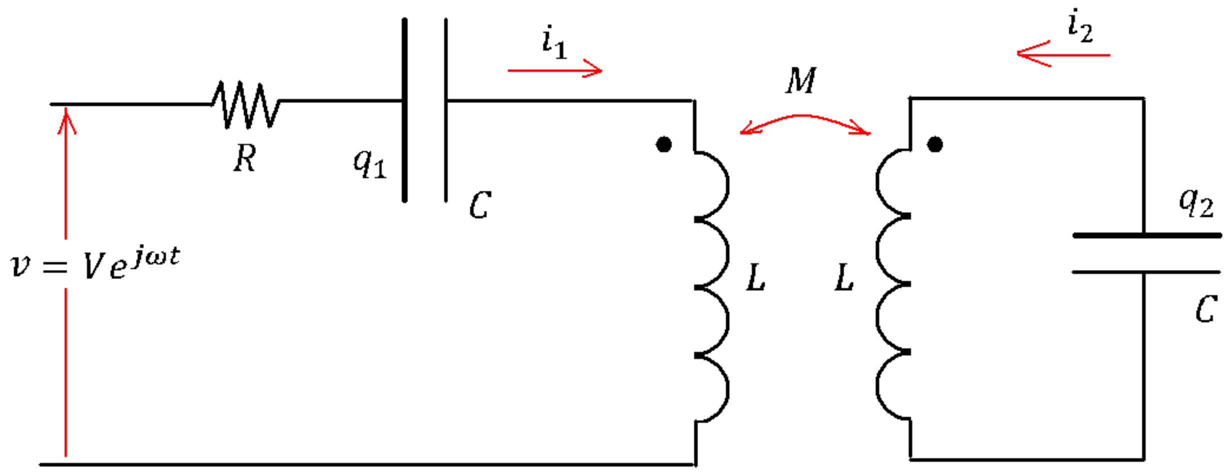
$$i_2 = -\frac{V}{M\left(\frac{R}{L} + \frac{1}{RC}\right)}\left(1 + j\frac{1}{R}\sqrt{\frac{L}{C}}\right)e^{j\omega t}$$

Because $\frac{dq_2}{dt} = -i_2$, the amplitude of q_2 is:

$$Q_2 = \frac{amplitude\ of\ i_2}{\omega} = \frac{V\sqrt{LC}}{M\left(\frac{R}{L} + \frac{1}{RC}\right)}$$

This amplitude shows us the right circuit's capacity to store energy transferred from the left circuit I suppose.

b. **<u>Case 2</u>: Connected in series:**



$$Ve^{j\omega t} = Ri_1 + \frac{q_1}{C} + L\frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{……………………(1)}$$

$$\frac{dq_1}{dt} = i_1 \quad \text{………………………………………………………(2)}$$

$$\frac{q_2}{C} = L\frac{di_2}{dt} + M\frac{di_1}{dt} \quad \text{……………………………………………(3)}$$

$$\frac{dq_2}{dt} = -i_2 \quad \text{……………………………………………………(4)}$$

From (1) and (2): $$j\omega Ve^{j\omega t} - RC\frac{di_1}{dt} - LC\frac{d^2i_1}{d^2t} - MC\frac{d^2i_2}{d^2t} = i_1$$

From (3) and (4): $$LC\frac{d^2i_2}{d^2t} + MC\frac{d^2i_1}{d^2t} = -i_2$$

Because all the currents are in sine form:

$$j\omega Ve^{j\omega t} - j\omega RCi_1 + \omega^2 LCi_1 + \omega^2 MCi_2 = i_1$$

$$-\omega^2 LCi_2 - \omega^2 MCi_1 = -i_2$$

Solve for i_1 with notice that $\omega = {}^1/_{\sqrt{LC}}$:

$$i_2 = -j\frac{V}{\omega M}e^{j\omega t}$$

Amplitude of q_2 is:

$$Q_2 = \frac{amplitude\ of\ i_2}{\omega} = \frac{V}{\omega^2 M} = \frac{VLC}{M}$$

I was surprised that this result doesn't contain R. That means, when resonance occurs in the series circuit case, R has no effect, i.e. heat loss on R is very small. This might mean that the energy stored in the series case is greater than that in the parallel case.

Indeed, some estimated values can show that. The other day at the lab, I noticed that the resonance frequency is $f \cong 600kHz$. The coil has about 20 rounds, is about 3cm high; its diameter is about 20cm. From that, I can estimate the scale of magnitude of L using the formula for solenoid's inductance, and thus, the capacitance C:

$$L = \mu_o N^2 \frac{A}{L} \approx 0.5mH$$

$$C = \frac{1}{4\pi^2 f^2 L} \approx 0.14\ nF$$

The internal resistance of the voltage source is $R \approx 50\ \Omega$ I suppose. Therefore, the ratio of Q_2 in the two cases is:

$$\frac{series}{parallel} \approx 38$$

That means, the efficiency of energy transfer in the series case is a lot greater. That's the reason why I don't understand why people connect the capacitor in parallel with the inductor, because our aim is to transfer energy from the primary coil to the secondary coil as much as possible. Is there any other reason that we have to sacrifice this? (e.g. the parallel method seems to be easier to control the energy flow, as it depends on R?) Or maybe there is something wrong with my calculation?