1) ∇r

R.C.S

$$\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\left(x^2+y^2+z^2\right)^{1/2}$$

$$\hat{x}\left(\tfrac{1}{2}\right)\left(x^2+y^2+z^2\right)^{-1/2}(2x)+\hat{y}\left(\tfrac{1}{2}\right)\left(x^2+y^2+z^2\right)^{-1/2}(2y)$$

$$+\ \hat{z}\left(\tfrac{1}{2}\right)\left(x^2+y^2+z^2\right)^{-1/2}(2z)$$

$$=\frac{(\hat{x}x+\hat{y}y+\hat{z}z)}{\sqrt{x^2+y^2+z^2}}=\frac{\vec{r}}{r}$$

C.C.S

$$\nabla r=\left(\hat{\rho}\frac{\partial}{\partial\rho}+\frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}+\hat{z}\frac{\partial}{\partial z}\right)\left(\rho^2+z^2\right)^{1/2}$$

$$=\hat{\rho}\left(\tfrac{1}{2}\right)\left(\rho^2+z^2\right)^{-1/2}(2\rho)+0+\hat{z}\left(\tfrac{1}{2}\right)\left(\rho^2+z^2\right)^{-1/2}(2z)$$

$$=\frac{\hat{\rho}\rho+\hat{z}z}{\sqrt{\rho^2+z^2}}=\frac{\vec{r}}{r}$$

S.C.S

$$\nabla r=\left(\hat{r}\frac{\partial}{\partial r}+\frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta}+\frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}\right)(r)$$

$$=\hat{r}\frac{\partial r}{\partial r}=\hat{r}=\frac{\vec{r}}{r}\qquad\boxed{\vec{r}=\hat{r}r}$$

2) $\nabla \frac{1}{r}$

R.C.S

$$\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\hat{x}\left(\frac{-1}{2}\right)\left(x^2+y^2+z^2\right)^{-3/2}(2x)+\hat{y}\left(\frac{-1}{2}\right)\left(x^2+y^2+z^2\right)^{-3/2}(2y)$$

$$+\hat{z}\left(-\frac{1}{2}\right)\left(x^2+y^2+z^2\right)^{-3/2}(2z)$$

$$=\frac{-\hat{x}x-\hat{y}y-\hat{z}z}{\left(\sqrt{x^2+y^2+z^2}\right)^3}=-\frac{\vec{r}}{r^3}$$

C.C.S

$$\nabla\frac{1}{r}=\left(\hat{\rho}\frac{\partial}{\partial\rho}+\frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}\ \hat{z}\frac{\partial}{\partial z}\right)\left(\rho^2+z^2\right)^{-1/2}$$

$$=\hat{\rho}\left(-\frac{1}{2}\right)\left(\rho^2+z^2\right)^{-3/2}(2\rho)+0+\hat{z}\left(\frac{-1}{2}\right)\left(\rho^2+z^2\right)^{-1/2}(2z)$$

$$=\frac{-\hat{\rho}\rho-\hat{z}z}{\left(\rho^2+z^2\right)^{3/2}}=-\frac{\hat{\rho}\rho+\hat{z}z}{\left(\rho^2+z^2\right)^{3/2}}=-\frac{\vec{r}}{r^3}$$

S.C.S

$$\nabla\frac{1}{r}=\left(\hat{r}\frac{\partial}{\partial r}+\frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta}+\frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}\right)(r)^{-1}$$

$$=-1\,\hat{r}\,r^{-2}=-\frac{\hat{r}}{r^2}=\frac{-\vec{r}}{r^3}\qquad \vec{r}=\hat{r}r$$

3) $\nabla \cdot \vec{r}$

R.C.S

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot (\hat{x}x + \hat{y}y + \hat{z}z)$$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$$

C.C.S

$$\left(\hat{\rho}\frac{\partial}{\partial \rho} + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}\right) \cdot (\hat{\rho}\rho + \hat{z}z)$$

$$\hat{\rho}\left(\frac{\partial}{\partial \rho}(\hat{\rho}\rho + \hat{z}z) + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial \phi}(\hat{\rho}\rho + \hat{z}z) + \hat{z}\frac{\partial}{\partial z}(\hat{\rho}\rho + \hat{z}\hat{z}\right)$$

$$\hat{\rho}\hat{\rho} = 1 \qquad \hat{\phi}\hat{\rho} = 0 \qquad \hat{\rho}\hat{z} = 0$$

$$\frac{\partial \rho}{\partial \rho} + \frac{\partial z}{\partial z} = 1 + 1 = 2$$

S.C.S

$$\nabla \cdot \vec{r} = \left(\hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial \phi} \right) \cdot (\hat{r}r)$$

multiply by $\frac{r^2}{r^2}$

$$\nabla \cdot \vec{r} = \frac{1}{r^2}\left(\hat{r}\frac{\partial}{\partial r}r^2 + \hat{\theta}r\frac{\partial}{\partial \theta} + \frac{r\hat{\phi}}{\sin\theta}\frac{\partial}{\partial \phi} \right) \cdot (\hat{r}r)$$

$$= \frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2 r) \right. = \frac{1}{r^2}\frac{\partial}{\partial r}r^3$$

$$= \frac{1}{r^2}(3r^2) = 3$$

S

$\nabla \times r$ R.C.S

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

$$\hat{x}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \hat{y}\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + \hat{z}\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right) = 0 - 0 + 0 = 0$$

C.C.S

$$\begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \rho & 0 & z \end{vmatrix} = 0$$

$$\hat{\rho}\left(\frac{1}{\rho}\frac{\partial z}{\partial \phi} - \frac{\partial 0}{\partial z}\right) - \hat{\phi}\left(\frac{\partial z}{\partial \rho} - \frac{\partial \rho}{\partial z}\right) + \hat{z}\left(\frac{\partial 0}{\partial \rho} - \frac{1}{\rho}\frac{\partial \rho}{\partial \phi}\right) = 0$$

S.C.S

$$\begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \\ r & 0 & 0 \end{vmatrix} = 0$$

$$\hat{r}\left(\frac{1}{r}\frac{\partial 0}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial 0}{\partial \phi}\right) - \hat{\theta}\left(\frac{\partial 0}{\partial r} - \frac{1}{r\sin\theta}\frac{\partial r}{\partial \phi}\right) + \hat{\phi}\left(\frac{\partial 0}{\partial r} - \frac{1}{r}\frac{\partial r}{\partial \theta}\right) = 0$$