

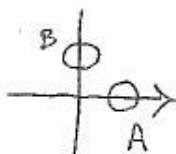
Transformations:

If a function has a geometric flavor, we call it a **transformation**. A set A is symmetric with respect to a transformation T if $T(A)=A$. Intuitively, this means that A is preserved by the transformation T .

Example 8: Let T be reflection through the y -axis in the xy -plane: $T(x, y) = (-x, y)$:



Then:

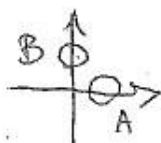


Thus: A is not symmetric with respect to T , while B is.

Exercise 8: Let S be reflection through the x -axis in the xy -plane: $S(x, y) = (x, -y)$:



Then:



Thus:

Example 9: Let R be reflection through the origin in the xy -plane: $R(x, y) = (-x, -y)$

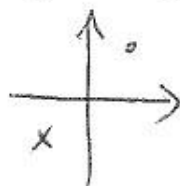


Then: $R(\{y = x^2 + 1\}) = \{(-y) = (-x)^2 + 1\} = \{-y = x^2 + 1\} = \{y = -(x^2 + 1)\}$. This suggests $\{y = x^2 + 1\}$ is NOT symmetric with respect to R . Because $(1, 2)$ is in the set, but $(-1, -2)$ is not, the set $\{y = x^2 + 1\}$ is NOT symmetric with respect to the transformation R .

But: $R(\{y^2 = x^2 + 1\}) = \{(-y)^2 = (-x)^2 + 1\} = \{y^2 = x^2 + 1\} = \{y = x^2 + 1\}$. This proves that the set $\{y^2 = x^2 + 1\}$ IS symmetric with respect to R .

(9)

Exercise 9: Let R be reflection through the origin in the xy -plane: $R(x, y) = (-x, -y)$



Then: $R(\{y^2 = x + 1\}) =$

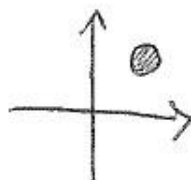
Then: $R(\{y^4 = x^2 + 1\}) =$

We can "make" a set A symmetric with respect to T by replacing A with the set:

$$\text{Symm}_T(A) = A \cup T(A) \cup T(T(A)) \cup T(T(T(A))) \cup \dots :$$

Example 10: Let T be rotation by a quarter turn around the origin:

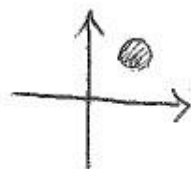
A :



$\text{Symm}_T(A)$:

Exercise 10: Let A be as below and: T reflection through y -axis, S reflection through x -axis, and R reflection through the origin of the xy -plane:

A :



$\text{Symm}_T(A)$:

$\text{Symm}_S(A)$:

$\text{Symm}_R(A)$:

