Velocity of a particle at time t in a rotating frame

Tags: rotating frame Edit

jamie.j1989 #1 Aug 31, 2018

OP 72 / 0

Imagine two frames one inertial (x,y,z) and the other rotating (x',y',z'), their origins are always coincident. The rotating frame is rotating as seen from the inertial frame with a time-dependent angular velocity $\boldsymbol{\Omega}(t)=(\Omega_x(t),\Omega_y(t),\Omega_z(t))$. In the rotating frame, a particle is moving in a circular fashion about the rotating frame's origin in the x-y plane at a radius r in the form $\boldsymbol{r}'(t)=r(\cos(\phi(t)),\sin(\phi(t)),0)$, where $\phi(t)$ is the angle between the position vector $\boldsymbol{r}'(t)$ in the rotating frame and the x' axis and $r=|\boldsymbol{r}'|$.

I would like to know what the velocity of the particle is as seen from the inertial frame at time t, I have started this by considering the relation between velocities of particles in two frames, one inertial the other rotating

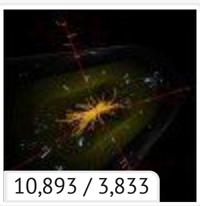
$$\frac{d\boldsymbol{r}}{dt}=\frac{d\boldsymbol{r}'}{dt}+\boldsymbol{\Omega}\times\boldsymbol{r}' \qquad (1)$$

Where the term on the R.H.S is the velocity of the particle as seen velocity of the particle as seen from the rotating frame, the second term being the rotation of the rotating frame as seen from the inertial frame.

I have some doubts about using this though, Eq.(1) seems to be an instantaneous velocity, it doesn't describe the evolution of the velocity as seen from the inertial frame for all times t. For example, if I give the position vector $\boldsymbol{r}'$ some time dependence $\boldsymbol{r}'(t)$ substitute into Eq.(1), what I get is the instantaneous velocity of the particle as if I started at the position $\boldsymbol{r}'(t)$? And similarly, giving a time dependence to $\boldsymbol{\Omega}$.

Or have I missed understood this equation?

vanhees71 #2 Aug 31, 2018



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I don't know, what bothers you. Your calculation is correct, and it describes the velocity in the inertial frame and relates it to the velocity in the rotating frame. What else are you looking for?

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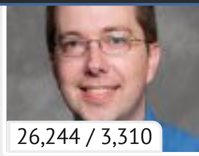
Dale #3 Aug 31, 2018

jamie.j1989 said: ↑



26,244 / 3,310

Insights Author

Staff: Mentor

> *seen from the inertial frame*

Why not? The evolution of the velocity in the inertial frame is:

$v(t) = \frac{dr(t)}{dt}$

jamie.j1989 #4 Aug 31, 2018

NEW

OP

72 / 0

> vanhees71 said: ↑
>
> *I don't know, what bothers you. Your calculation is correct, and it describes the velocity in the inertial frame and relates it to the velocity in the rotating frame. What else are you looking for?*

Specifically I am trying to look at the difference of the absolute velocities squared of two particles, one going clockwise (-) around a circuit in the rotating-frame and the other anticlockwise (+). When plugging in for $\boldsymbol{r}'_{\pm} = r(\cos(\phi(t)), \pm\sin(\phi(t)), 0)$ into Eq.(1) I am not getting something I can make sense of when thinking about particular scenarios. Explicitly, I am calculating

$$|\dot{\boldsymbol{r}}_{\pm}|^2 = \left|\dot{\boldsymbol{r}}'_{\pm}\right|^2 + 2\dot{\boldsymbol{r}}'_{\pm} \bullet \Omega \times \boldsymbol{r}'_{\pm} + \left|\boldsymbol{\Omega} \times \boldsymbol{r}'_{\pm}\right|^2 \qquad (2)$$

And looking at $|\dot{\boldsymbol{r}}_{+}|^2 - |\dot{\boldsymbol{r}}_{-}|^2$.

$$|\dot{\boldsymbol{r}}_{+}|^2 - |\dot{\boldsymbol{r}}_{-}|^2 = \Omega_x\Omega_y \sin(2\phi) - 2\dot{\phi}\Omega_z \qquad (3)$$

However, as FactChecker has mentioned, if Ω is time

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scenario, which is, if I initially only have rotation about the x-axis which comes to a halt when a 90 deg rotation has occurred and at this point the two particles have also gone through a 90 deg rotation about the circuit, which puts them on the z-axis of the inertial frame one directly below the other, if at this point I have rotation about the z-axis I would expect the difference above to be 0, I do not see this scenario in the above equation.

So considering that was a scenario having time dependence in Ω I need to consider a product term on $\boldsymbol{\Omega} \times \boldsymbol{r}$?

Last edited: Aug 31, 2018

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jamie.j1989 #5 Sunday at 8:48 AM

OP
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Would the correct equation to use for time-dependent $\boldsymbol{\Omega}$ be

$$\boldsymbol{a} = \boldsymbol{a}' + 2\boldsymbol{\Omega} \times \boldsymbol{v}' + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}') + \frac{d\boldsymbol{\Omega}}{dt} \times \boldsymbol{r}' \quad (4)$$

Where the $\boldsymbol{a}$'s are now the acceleration in the respective frames. And then integrate with respect to time to solve for the $\boldsymbol{v}$'s?

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FactChecker #6 Sunday at 9:33 AM

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The equation for acceleration does not make your original equation for velocity wrong. It seems like you are concerned that your original equation is wrong when the rotation rate is a function of time. But that does not make the instantaneous

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vanhees71 likes this.

jamie.j1989 #7 Sunday at 9:51 AM

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Yes, I am concerned that it is wrong when having a time-dependent rotation, this is because the result I get Eq.(3), doesn't seem to make sense when I consider particular examples of rotation with a time dependence. Admittedly this could just be a mistake in my workings or reasoning about the example I'm going through. I think I will go over my workings on this and try to make sense of it a bit more.

Dale #8 Sunday at 11:33 AM

26,244 / 3,310
Insights Author
Staff: Mentor

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Both your equation in post 1 and in post 5 are good. Of course, Ω should be continuous.

FactChecker #9 Sunday at 11:44 AM

NEW

Of course, you must always be careful in this to distinguish between an acceleration due to a force and a derivative of velocity wrt time. They are not the same.

straight and level, unaccelerated flight. There is no force-caused acceleration in any frame because there is no force. But if we looked at a pilot head-centered-head-fixed coordinate system, there would be a huge $velocity_{head-centered-head-fixed}$ time derivative any time he turned his head. That velocity could go from 100 MPH in the direction of his nose to 0 MPH in a second if he turns his head to the side.

Dale likes this.

jamie.j1989 #10 Monday at 10:08 AM

Ok, so I've convinced myself that the result is fine. However, im now confused about coordinate transformations. If I write $\dot{\boldsymbol{r}} = \boldsymbol{v}$ as

$$\boldsymbol{v} = v_{x'}\hat{\boldsymbol{i}}' + v_{y'}\hat{\boldsymbol{j}}' + v_{z'}\hat{\boldsymbol{k}}' \tag{5}$$

and I want to look at the velocity component $v_z\hat{\boldsymbol{k}}$ in the inertial frame I have

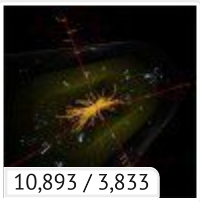
$$v_z = v_{x'}\hat{\boldsymbol{i}}' \bullet \hat{\boldsymbol{k}} + v_{y'}\hat{\boldsymbol{j}}' \bullet \hat{\boldsymbol{k}} + v_{z'}\hat{\boldsymbol{k}}' \bullet \hat{\boldsymbol{k}} \tag{6}$$

I'm confused about how to evaluate the projection terms, for example, my first attempt was to just put $\hat{\boldsymbol{i}}'$ into Eq.(1) and go from there, but doing that gives me

$$\dot{\hat{\boldsymbol{i}}}' = \Omega_z\hat{\boldsymbol{j}}' - \Omega_y\hat{\boldsymbol{k}}' \tag{7}$$

which just seems circular?

vanhees71 #11 Wednesday at 4:34 AM



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The primed and unprimed basis vectors are related by (time dependent!) rotations. Let me switch notation and use the Einstein summation convention. The basis vectors fixed to the inertial frame are time-independent by definition, while the basis vectors in the rotating frame are timedependent, and the rotation matrix is given by

$$\vec{e}'_j(t) \cdot \vec{e}_k = D_{jk}(t).$$

Now let's calculate the time derivative of an arbitrary vector $\vec{V}(t)$. We have

$$\vec{V}(t) = V_k(t)\vec{e}_k = V'_j(t)\vec{e}'_j(t) = V'_j(t) D_{jk}(t)\vec{e}_k.$$

Then you get

$$\dot{\vec{V}}(t) = \dot{V}_k(t)\vec{e}_k = [\dot{V}'_j(t) D_{jk}(t) + V'_j(t)\dot{D}_{jk}(t)]\vec{e}_k = \dot{V}'_j(t)$$

Now

$$D_{jk}(t) D_{jl}(t) = \delta_{jl} \ \Rightarrow \ \omega'_{jl} = \dot{D}_{jk}(t) D_{jl}(t) = -D_{jk} \cdot \dot{D}_{jl}(t$$

Now we can write

$$\omega'_{lj}(t) = \epsilon_{lmj} \Omega'_m(t).$$

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This leads finally to

$$\dot{\vec{V}} = [\dot{V}'_l(t) + \epsilon_{lmj}\Omega'_m(t)V'_j(t)]\vec{e}'_l.$$

Now using the matrix notation, with bold symbols standing for column $\mathbb{R}^3$ vectors, you can define the "covariant time derivative" in the rotating frame as

$$\mathrm{D}_t \boldsymbol{V}' = \dot{\boldsymbol{V}}' + \boldsymbol{\Omega}' \times \boldsymbol{V}'.$$

For the position vector you get

$$\boldsymbol{v}' = \mathrm{D}_t \boldsymbol{r}' = \dot{\boldsymbol{r}}' + \boldsymbol{\Omega}' \times \boldsymbol{r}'.$$

For the acceleration you finally find

$$\boldsymbol{a}' = \mathrm{D}_t \boldsymbol{v}' = \dot{\boldsymbol{v}}' + \boldsymbol{\Omega}' \times \boldsymbol{v}' = \ddot{\boldsymbol{r}}' + 2\boldsymbol{\Omega}' \times \dot{\boldsymbol{r}}' + \boldsymbol{\Omega}' \times (\boldsymbol{\Omega}' \times \boldsymbol{r}') +$$

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weirdoguy likes this.

jamie.j1989 #12 Wednesday at 9:55 AM

OP

72 / 0

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If I understand correctly what you have shown, and if I can write it down in a way I find easier to read, I have

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v'_x \\ v'_y \\ v'_z \end{pmatrix} + \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} r'_x \\ r'_y \\ r'_z \end{pmatrix}$$

And so what I'm looking for is just

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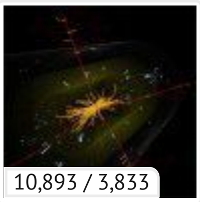
vanhees71 #13 Yesterday at 7:09 AM



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No (8) is not correct. There's always a rotation matrix between the primed and unprimed components, and your angular-velocity tensor must get primed indices. As you can see from the 2nd formula in #11 for any vector $\vec{V}$ (symbols with arrows are the invariant vectors) you have (bold symbols are the components wrt. the inertial basis (unprimed) or the rotating basis (primed)). From the said formula you get

$$\boldsymbol{V}=\hat{D}^{\text{T}}\boldsymbol{V}' \;\Rightarrow\; \boldsymbol{V}'=\hat{D}\boldsymbol{V}.$$

Thus for the velocity components you get

$$\boldsymbol{v}=\hat{D}^{\text{T}}\boldsymbol{v}'=\hat{D}^{\text{T}}(\dot{\boldsymbol{r}}'+\boldsymbol{\Omega}'\times\boldsymbol{r}').$$

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