Vibrations of a Bucket Lift

For the sake of this analysis, the boom of the lift is simplified to be a uniform beam. In the original problem as posted on PF, the nonuniform nature of the telescoping boom was certainly a major point, but in the process, the fundamentals of the analysis seem to have been obscured. Getting back to those fundamentals is the objective of replacing the boom with a uniform beam. Note that the beam has properties as noted,

L = length of the boom

A = cross sectional area of the boom

I = area moment of inertia for boom cross section

E = Young's modulus for boom material

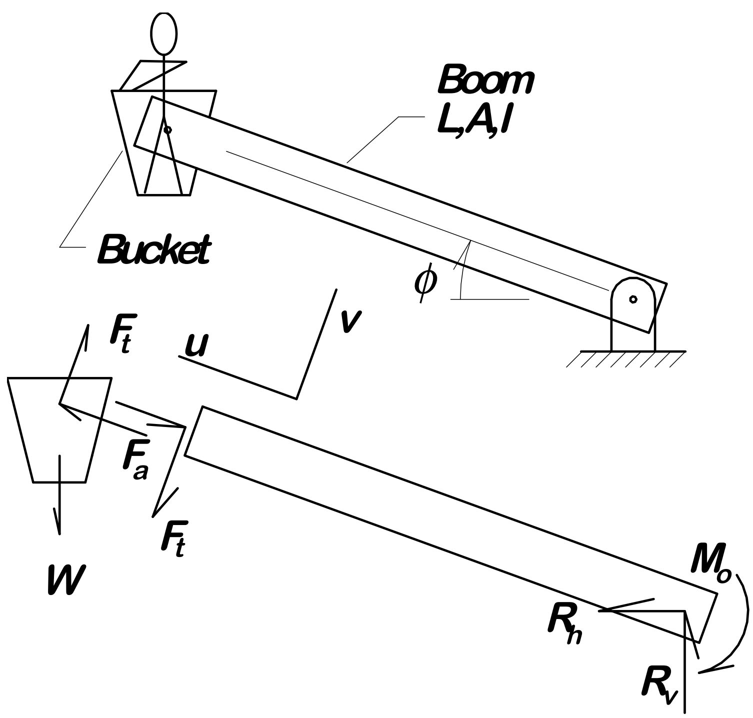

Figure 1: Bucket Lift

In the adjoining figure, the upper view is a pictorial representation of the system while the lower part presents free body diagrams (FBDs) for the boom and the bucket. In considering the FBD for the bucket, it is convenient to resolve the forces on a coordinate system parallel and perpendicular to the boom axis, denoted as $u - v$. Writing Newton's Second Law for the bucket (with the man, considered as a single rigid body (**assumption**)), the equations of motion are

$$\frac{W}{g}\ddot{u} = \sum F_u = F_a - W\sin\phi \tag{1}$$

$$\frac{W}{g}\ddot{v} = \sum F_v = F_t - W\cos\phi \tag{2}$$

If the mass of the beam is neglected (**assumption**), the beam is nothing more than a cantilever beam with transverse and axial tip loads. The force–deflection relations for such a beam are well known from Mechanics of Materials:

$$u = -\frac{F_a L}{AE} \quad \text{axial displacement} \tag{3}$$

$$v = -\frac{F_t L^3}{3EI} \quad \text{bending displacement} \tag{4}$$

These are solvable for the axial and transverse stiffnesses:

$$K_a = -\frac{AE}{L} \tag{5}$$

$$K_t = -\frac{3EI}{L^3} \tag{6}$$

The stiffnesses can be used to replace the force terms in the equations of motion:

$$\frac{W}{g}\ddot{u} + \frac{AE}{L}u = W\sin\phi \tag{7}$$

$$\frac{W}{g}\ddot{v} + \frac{3EI}{L^3}v = W\cos\phi \tag{8}$$

These are linear ordinary differential equations of motion, but they are not homogeneous. Consider first the particular (static) solution to each one:

$$u = u_o = \text{constant} \tag{9}$$

$$v = v_o = \text{constant} \tag{10}$$

For this solution we get

$$u_o = \frac{WL}{AE}\sin\phi \tag{11}$$

$$v_o = \frac{WL^3}{3EI}\cos\phi \tag{12}$$

These expressions give the static deformation of the lift under the weight W when the boom angle is ϕ, but they do not describe the vibratory motion. To get to the vibrations, consider a transformation of variables:

$$u = u_o + U \tag{13}$$
$$v = V_o + V \tag{14}$$

where U and V are the dynamic, vibratory displacements in the u and v directions. Substituting these transformations into the equations of motion gives

$$\frac{W}{g}\ddot{u} + \frac{AE}{L}u = \frac{W}{g}\ddot{U} + \frac{AE}{L}(u_o + U) = W\sin\phi \tag{15}$$
$$\frac{W}{g}\ddot{v} + \frac{3EI}{L^3}v = \frac{W}{g}\ddot{V} + \frac{3EI}{L^3}(v_o + V) = W\cos\phi \tag{16}$$

When the values of u_o and v_o are used, this reduces to

$$\frac{W}{g}\ddot{U} + \frac{AE}{L}U = 0 \tag{17}$$
$$\frac{W}{g}\ddot{V} + \frac{3EI}{L^3}V = 0 \tag{18}$$

These are the expected forms for a free vibration. Note that there are two different natural frequencies defined:

$$\omega_{axial} = \sqrt{\frac{AEg}{WL}} \tag{19}$$
$$\omega_{transverse} = \sqrt{\frac{3EIg}{WL^3}} \tag{20}$$

For all realistic system parameter values, it will be found that $\omega_{axial} \gg \omega_{transverse}$. Note also that the angle ϕ only serves to define the direction of the vibratory motion; it has no effect on the natural frequency.

The same result will be obtained if the equivalent stiffness of the stepped boom is substituted for K_a in the development above.