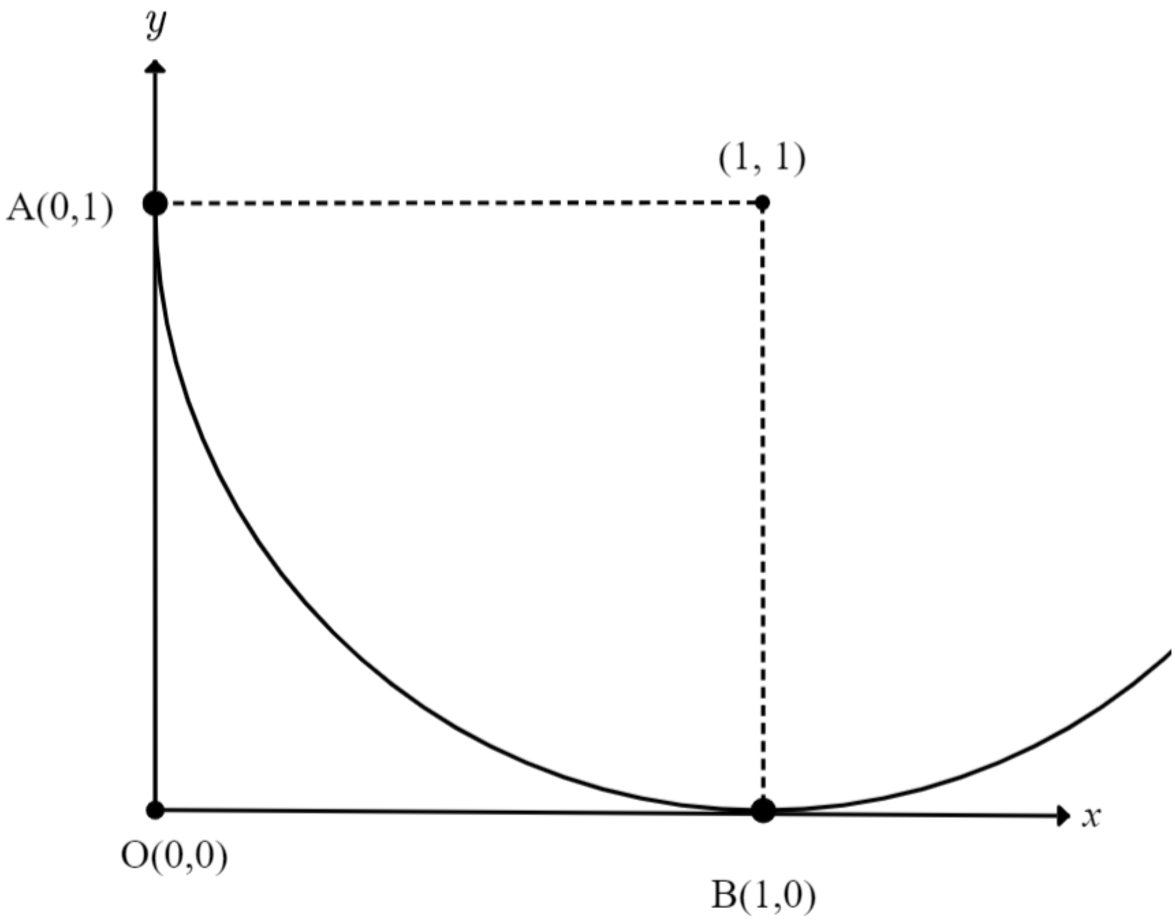
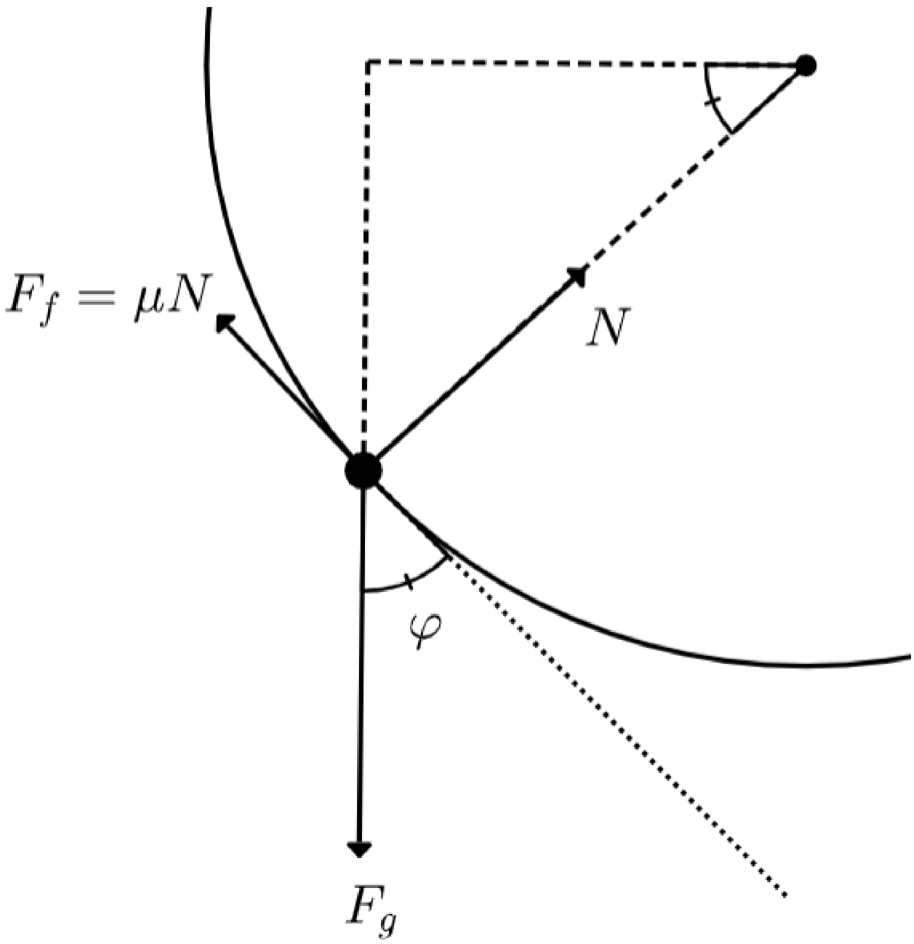

y
(1, 1)
A(0,1)
x
O(0,0)
B(1,0)


$F_f = \mu N$
N
φ
F_g

φ is the angle formed between the tangent to the circle and the vertical (see the free body diagram). The circle has radius 1, so:

Position vector of the mass $= \vec{p} = \begin{pmatrix} 1 - \cos\varphi \\ 1 - \sin\varphi \end{pmatrix}$

Differentiating w.r.t. time:

$$\vec{v} = \begin{pmatrix} \dot{\varphi}\sin\varphi \\ -\dot{\varphi}\cos\varphi \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} \ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi \\ -\ddot{\varphi}\cos\varphi + \dot{\varphi}^2\sin\varphi \end{pmatrix} = \begin{pmatrix} \ddot{\varphi}\sin\varphi \\ -\ddot{\varphi}\cos\varphi \end{pmatrix} + \begin{pmatrix} \dot{\varphi}^2\cos\varphi \\ \dot{\varphi}^2\sin\varphi \end{pmatrix}$$

$\vec{a}$ consists of a tangential component (magnitude $m\ddot{\varphi}$) and a normal / centripetal component ($m\dot{\varphi}^2$ because the radius is 1). Honestly, the above bit is unnecessary, but it might make my actual working clearer. In short, I have to prove every single dang physics thing I use in this report except F = ma (long story and you're probably not that interested).

Summing the forces in the normal and tangential directions produces a system of equations:

Normal: $mgsin\varphi - \mu N = m\ddot{\varphi} \Longrightarrow N = \frac{mgsin\varphi - m\ddot{\varphi}}{\mu}$

Tangential: $N - mgcos\varphi = \dot{\varphi}^2 \Longrightarrow N = \dot{\varphi}^2 + mgcos\varphi$

By the arclength formula, the distance travelled by the mass is equal to the angle φ (because the radius is 1, once again):

$$s = \varphi$$

Hence, eliminating N produces a differential equation dependent on time:

$$\mu\left(\frac{ds}{dt}\right)^2 + \mu mgcos(s) = mgsin(s) - m\frac{d^2s}{dt^2}$$

Which doesn't seem to have a very nice solution.

This might all be nonsense – then again, I wouldn't be asking for help if I knew how to do it. I tried to find s(t) but my approach does not resolve itself very nicely (see above). A more efficient strategy would be welcomed (but energy isn't allowed!).

Thanks,
Ed