The origin of Casimir effect: Vacuum energy or van der Waals force?

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Outline:

Part 1. Introduction

Part 2. Proof that Casimir force does not originate from vacuum energy of electromagnetic field

Part 3. The physical origin of Casimir effect - qualitative picture

Part 4. A toy model for Casimir-like effect

Part 1.
INTRODUCTION

Motivation and spoilers:

Frequent statement 1:
- In physics we measure energy differences, not absolute energies. Vacuum energy is therefore unphysical and can be removed by a simple subtraction.

Frequent statement 2:
- Casimir effect demonstrates that vacuum energy is physical.

Contradiction? How can both statements be true?

Spoiler:
The word “vacuum” in statement 1
does not have the same meaning as “vacuum” in statement 2.

(Details later in the talk.)

In the literature, two explanations of Casimir force:
1) vacuum energy of electro-magnetic field
2) van der Waals force
- Which explanation is correct?

> *So, in the discussion session after Casimir's lecture I switched topic and asked: "Is the Casimir effect due to the quantum fluctuations of the electromagnetic field, or is it due to the van der Waals forces between the molecules in the two media?" Casimir's answer began, "I have not made up my mind."*
>
> (I.H. Brevik, from the Foreword in S.Y. Buhmann, *Dispersion Forces I* (Springer-Verlag, Berlin, 2012).)

The goal of this talk is to resolve such conceptual puzzles about Casimir effect.

Based on my 2 papers:

1) H.N., Phys. Lett. B **761**, 197 (2016)
- no go theorem, what the microscopic origin of Casimir force is **not**.

2) H.N., Ann. Phys. **383**, 181 (2017)
- says what the microscopic origin of Casimir effect **is**.

Spoiler:
The main conclusions will be that
1) van der Waals forces give a **fundamental microscopic** description.
2) Vaccum energy approach is an **effective macroscopic** description.

Historical aspects:

- J.D. van der Waals (1873) introduced intermolecular forces **phenomenologically**, without theoretical explanation.

- F. London (1930) gave first explanation of intermolecular forces (also called London forces) in terms of **nonrelativistic QM**.

- H.B.G. Casimir and D. Polder (1948) found a simple expression for intermolecular forces with **relativistic** effects (retardation) included.
- Intrigued by simplicity of the result, Casimir searched for a **simpler explanation**.

- N. Bohr suggested to Casimir that it should be somehow related to **vacuum energy**.
- This inspired Casimir to do a simpler calculation, now based on vacuum energy instead od van der Waals forces.
- Finally, Casimir found (1948) that calculation based on vacuum energy further simplifies when molecules are replaced by **perfectly conducting plates** - this is the calculation that can be found in texbooks.

- E.M. Lifshitz (1956) found general theory for computing van der Waals forces between plates which are **non-perfect conductors** and shown that Casimir force emerges as a special case.

- J. Schwinger (1975) found another way to compute Casimir force without referring to vacuum energy.

Confusion in modern literature:

- One culprit for confusion are **high-energy** physics textbooks.
- Naively, one would expect that they emphasize the microscopic origin.
- Yet, typical general high-energy (particle physics and gravity) textbooks talk only about the vacuum-energy origin of Casimir force:

- C. Itzykson, J-B. Zuber, *Quantum Field Theory* (1980).
  (discusses van der Waals approach only in fine print.)
- N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space* (1982).
- J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (2002).
- V. Mukhanov, S. Winitzki, *Introduction to Quantum Effects in Gravity* (2007).
- A. Zee, *Quantum Field Theory in a Nutshell* (2010).
- M.D. Schwartz, *Quantum Field Theory and the Standard Model* (2014).
- T. Padmanabhan, *Quantum Field Theory* (2016).
  (mentions van der Waals approach only in a footnote.)

- Another culprit for confusion are **condensed-matter** textbooks.
- van der Waals description of Casimir force is a condensed matter effect.
- Yet, general condensed-matter textbooks usually don't discuss Casimir effect at all.

General texbooks that emphasize the van der Waals origin of Casimir force:
- E.M. Lifshitz, L.P. Pitaevskii, *Statistical Physics: Part 2* (1980).
  or **1st edition** of L.D. Landau, E.M. Lifshitz,
  Electrodynamics of Continuous Media.
- E.D. Commins, *Quantum Mechanics* (2014).

There are also many specialized monographs that treat both approaches to Casimir effect. I was particularly influenced by:
- V.A. Parsegian, *Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists* (2006).
- S.Y. Buhmann, *Dispersion Forces I and II* (2012).
- W.M.R. Simpson, *Surprises in Theoretical Casimir Physics* (2015).

Pragmatic point of view:

- In specialized literature, two approaches to Casimir force usually considered as two **complementary methods**.
- The issue is which method is more practical, not which approach is "more true".
- Eventually, two methods give the same results.

$\Rightarrow$ No controversy from practical point of view.

Conceptual point of view:

- Controversy exists only from conceptual point of view.
- Not many papers on conceptual aspects.

Best known paper on conceptual aspects:
- R.L. Jaffe, Phys. Rev. D **72**, 021301 (2005);
hep-th/0503158; ~ 150 citations (inSPIRE).
- Concludes that Casimir force is manifestation of van der Waals forces, which can be calculated without referring to vacuum energy.

With motivation to further reduce conceptual confusion,
I have published two papers with conclusions similar to that of Jaffe:
H.N., Phys. Lett. B **761**, 197 (2016)
H.N., Ann. Phys. **383**, 181 (2017)

Part 2.

PROOF THAT CASIMIR FORCE DOES NOT ORIGINATE FROM VACUUM ENERGY OF ELECTROMAGNETIC FIELD

Heuristic idea:

Energy of electromagnetic field

$$H_{\text{em}} = \int d^3x \, \frac{\mathbf{E}^2 + \mathbf{B}^2}{2}$$

In general, Fourier transform contains all wave vectors $\mathbf{k}$

$$\mathbf{E}(\mathbf{x}) = \int d^3k \ldots$$

However, Maxwell equations $\Rightarrow$ $\mathbf{E}$ vanishes at perfect conductor
$\Rightarrow$ boundary conditions for $\mathbf{E}$
If two conducting plates separated by distance z $\Rightarrow$
contributions in z-direction only from

$$k_z = n\pi/z \ \text{ for } \ n = 1, 2, 3, \ldots$$
$$\int d^3k \to \int d^2k \sum_{k_z}$$

$\Rightarrow$

$$\mathbf{E} \to \tilde{\mathbf{E}}(z), \ \ \mathbf{B} \to \tilde{\mathbf{B}}(z)$$
$$H_{\text{em}} \to \tilde{H}_{\text{em}}(z)$$

Now the vaccum energy depends on z

$$\tilde{E}_{\text{vac}}(z) = \langle 0|\tilde{H}_{\text{em}}(z)|0\rangle$$

The force

$$\tilde{F}(z) = -\frac{\partial \tilde{E}_{\text{vac}}(z)}{\partial z}$$

Standard textbook calculation (e.g. Itzykson and Zuber)

$$\tilde{E}_{\text{vac}}(z) = L^2 \int \frac{d^2k}{(2\pi)^2} \sum_{k_z} \sum_{\text{polariz's}} \frac{\hbar\omega}{2}$$

The final result - force per area:

$$L^{-2}\tilde{F}(z) = -\frac{\pi^2}{240}\frac{\hbar c}{z^4}$$

- in agreement with measurements.

However, the expression for the force is valid only if z itself is dynamical, e.g.

$$\tilde{H} = \frac{p_z^2}{2m} + \tilde{E}_{\text{vac}}(z)$$

where $m = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the plates.

But we started from H_{em} which does not depend on z, suggesting

$$F = -\frac{\partial \langle 0|H_{\text{em}}|0\rangle}{\partial z} = 0$$

- The fundamental Hamiltonian is H_{em}, not $\tilde{H}_{\text{em}}(z)$.
- $\tilde{H}_{\text{em}}(z)$ has implicit (not explicit) dependence on z.
- Hamilton equations of motion require explicit dependence.
$\Rightarrow$ suggests that calculation of $\tilde{F}(z)$ is kind of cheating.
- The final result is correct, but the conceptual picture emerging from such calculation is misleading.

The main proof:

Full action of quantum electrodynamics (QED):

$$I = I_{\text{em}}(A) + I_{\text{matt}}(\phi) + I_{\text{int}}(A, \phi)$$

where $A(x) = \{A^\mu(x)\}$ is the EM field, $\phi(x)$ denotes all matter fields,

$$I_{\text{em}}(A) = -\frac{1}{4} \int d^4x \, F^{\mu\nu} F_{\mu\nu}$$

$$I_{\text{int}}(A, \phi) = - \int d^4x \, A_\mu j^\mu(\phi)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $j^\mu(\phi)$ is the charge current. $\Rightarrow$

$$H = H_{\text{em}}(A, \pi_A) + H_{\text{matt}}(\phi, \pi_\phi) + H_{\text{int}}(A, \phi, \pi_\phi)$$

Heisenberg equations of motion for matter:

$$\dot{\phi} = i[H, \phi], \quad \dot{\pi}_\phi = i[H, \pi_\phi]$$

$H_{\text{em}}(A, \pi_A)$ does not have explicit dependence on ϕ and π_ϕ, so

$$[H_{\text{em}}, \phi] = 0, \quad [H_{\text{em}}, \pi_\phi] = 0$$

$\Rightarrow$

$$\dot{\phi} = i[H_{\text{int}}, \phi] + i[H_{\text{matt}}, \phi],$$
$$\dot{\pi}_\phi = i[H_{\text{int}}, \pi_\phi] + i[H_{\text{matt}}, \pi_\phi].$$

$\Rightarrow$ H_{em} **does not contribute to the quantum force on matter**.

Now **suppose** that EM field is in the vacuum state $|0_A\rangle$.
This means that the full state has the form

$$|\Omega\rangle = |0_A\rangle|\psi_\phi\rangle$$

where $|\psi_\phi\rangle$ is the state of matter, e.g. Casimir plates.

But $\langle 0_A|A^\mu|0_A\rangle = 0$ and H_{int} is **linear** in A^μ, so

$$\langle\Omega|H_{\text{int}}|\Omega\rangle = 0$$

H_{em} is quadratic in A^μ, so all EM vacuum energy comes from

$$E_{\text{vac}} = \langle\Omega|H_{\text{em}}|\Omega\rangle = \langle 0_A|H_{\text{em}}|0_A\rangle.$$

So, (i) only H_{em} contributes to the EM vacuum energy,
but (ii) H_{em} does not contribute to quantum forces on matter.
Hence, **EM vacuum energy does not contribute to quantum forces on matter**.

Physical interpretation: The Casimir force comes from H_{int},
but to get $\langle\Omega|H_{\text{int}}|\Omega\rangle \neq 0$ one must have $|\Omega\rangle \neq |0_A\rangle|\psi_\phi\rangle$.

Part 3.

THE PHYSICAL ORIGIN OF CASIMIR EFFECT - QUALITATIVE PICTURE

- Spectrum of h.o.

$$E_n = \hbar\omega \left(n + \frac{1}{2}\right)$$

$\Rightarrow$ Energy of the ground state $E_0 = \hbar\omega/2$.
- Is this energy physical?

- Standard answer - no, because we only measure energy **differences**.
$\Rightarrow$ We can subtract this constant without changing physics

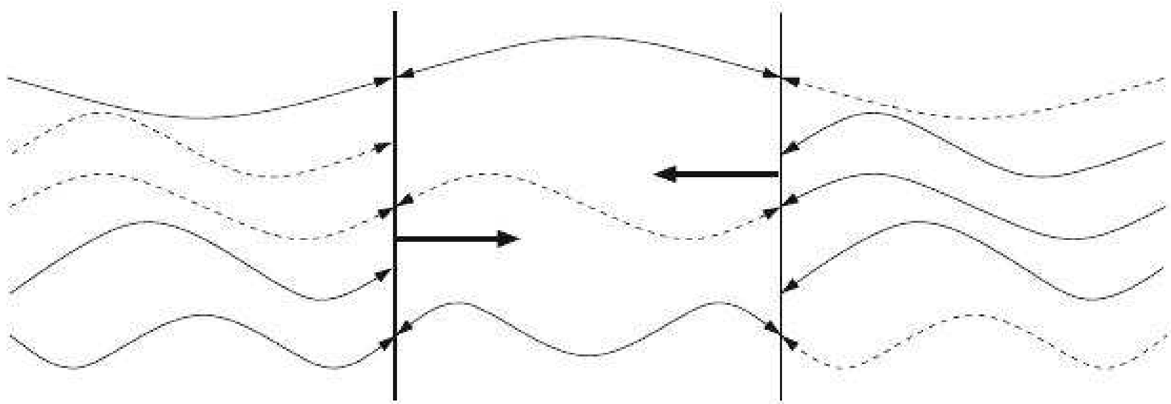
$$\Rightarrow \quad E_n = \hbar\omega n$$

- On the other hand, often claimed in literature that Casimir effect is a counter-example.
- Is Casimir effect evidence that vacuum energy is physical?

Casimir effect = attractive force between electrically neutral plates

Two explanations:
1) vacuum energy of electro-magnetic field
2) van der Waals force
- Which explanation is correct?

1) Vacuum-energy explanation:



- field vanishes at perfectly conducting plates

$\Rightarrow$ some wavelengths impossible between the plates

$\Rightarrow$ Hamiltonian does not contain those modes

$\Rightarrow$ those modes do not contribute to vacuum energy E_{vac}

$\Rightarrow$ E_{vac} depends on the distance z between the plates

$\Rightarrow$ Casimir force

$$F_{\text{vac}} = -\frac{\partial E_{\text{vac}}(z)}{\partial z} = -L^2 \frac{\pi^2}{240} \frac{\hbar c}{z^4}$$

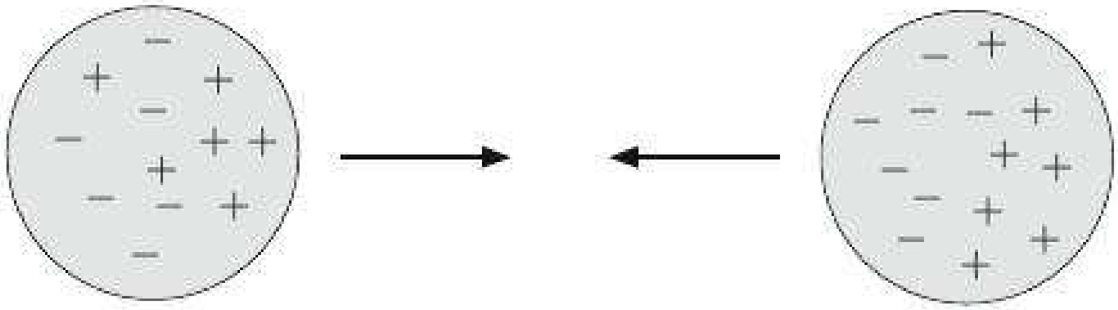
Advantages:
- calculation relatively simple
- presented in many textbooks

Disadvantages:
- Electromagnetic forces are forces between charges,
but where are the charges?
- Force originates from boundary conditions, but microscopic origin
of boundary conditions not taken into account.
$\Rightarrow$ Vacuum-energy explanation is not a fully microscopic explanation.

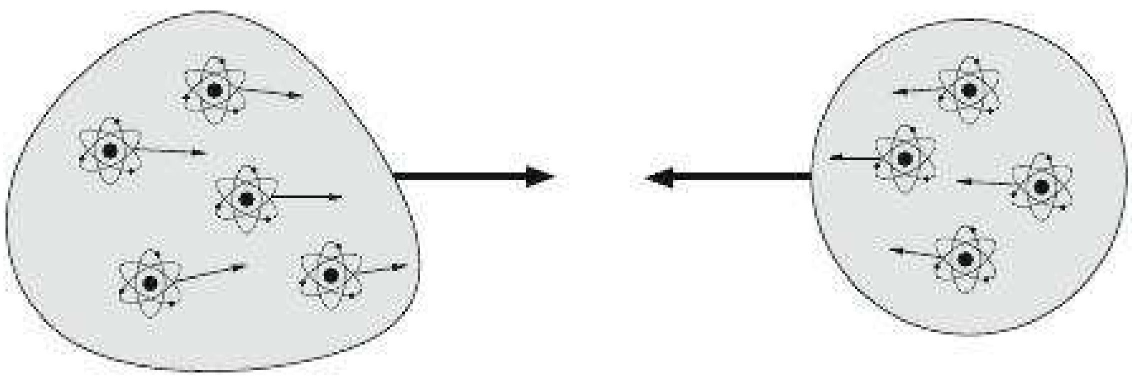
Those disadvantages avoided by van der Waals force approach.

2) Van der Waals force explanation:

- Force explained by polarization of the medium:

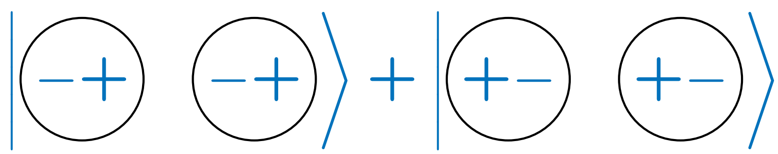


- Polarizability of the medium traced down to microscopic polarizability of atoms:



- Clasically, there is no spontaneous polarization because 2 molecules cannot decide should they polarize as

- Quantum mechanically they do not need to decide because they can be in superposition



$\Rightarrow$ van der Waals force is a quantum effect.

- Semiclassical intuitive picture of superposition:
the system "fluctuates" between two classical polarizations.

- Calculation for metal plates quite complicated (Lifshitz theory).
- The final result is the same $F_{\text{vdW}} = F_{\text{vac}}$.

Why do two different explanations give the same result?

Qualitative explanation:

- vacuum-energy explanation originates from boundary conditions
- boundary conditions originate from $\mathbf{E} = 0$ in a perfect conductor
- $\mathbf{E} = 0$ originates from rearrangement of charges

so that any external $\mathbf{E}_{\text{ext}}$ is canceled

- rearrangement of charges $=$ polarization $\mathbf{P}(\mathbf{x})$

(electric dipole moment per volume)

- such a system is simpler to describe by electric displacement

$$\mathbf{D} = \mathbf{E} + \mathbf{P}$$

- $\mathbf{P}$ is induced by $\mathbf{E}$, so approximately $\mathbf{P} \propto \mathbf{E}$

$\Rightarrow$ $\mathbf{D} = \epsilon \mathbf{E}$ (ϵ is dielectric constant) $\Rightarrow$

$$\mathbf{P} = (\epsilon - 1)\mathbf{E}$$

- energy density in dielectric medium (Jackson, *Classical Electrodyn.*)

$$\mathcal{H} = \frac{\mathbf{D} \cdot \mathbf{E}}{2}$$

- combining all the equations above $\Rightarrow$

$$\mathcal{H} = \frac{\mathbf{E}^2}{2} + \frac{\mathbf{P} \cdot \mathbf{E}}{2}$$

- assume there is no external electric field $\Rightarrow$ average field vanishes, i.e.

$$\langle \mathbf{E} \rangle = \langle \mathbf{P} \rangle = 0$$

- however there are quantum fluctuations $\langle \mathbf{E}^2 \rangle \neq 0 \Rightarrow$

$$\boxed{\langle \mathcal{H}_{\text{int}} \rangle = \frac{\langle \mathbf{P} \cdot \mathbf{E} \rangle}{2} = \frac{\langle \mathbf{P}^2 \rangle}{2(\epsilon - 1)} = \frac{\epsilon - 1}{2} \langle \mathbf{E}^2 \rangle}$$

$\Rightarrow$ interaction energy originates from correlation $\langle \mathbf{P} \cdot \mathbf{E} \rangle$

- this is van der Waals energy
- this is **fundamental** because it does not depend on phenomenological macroscopic parameter ϵ.

At a **phenomenological** macroscopic ϵ-dependent level, can also be interpreted as:

- energy of polarization fluctuations $\langle \mathbf{P}^2 \rangle$, or
- energy of electric field fluctuations $\langle \mathbf{E}^2 \rangle$ (the "vacuum"-energy description of Casimir effect)

Relation between perfect and real conductors:

$\mathbf{E} = 0$ inside the perfect conductor
$\Rightarrow$ boundary conditions: no modes with $\mathbf{k}$ for which $k_z \neq n\pi/z$
$\Rightarrow$ frequencies $\omega_{\mathbf{k}}$ with $k_z \neq n\pi/z$ do not contribute

Equivalently, we can say that those frequencies are zero:

$$\omega_{\mathbf{k}} = \begin{cases} c|\mathbf{k}| & \text{for } k_z = n\pi/z, \\ 0 & \text{for } k_z \neq n\pi/z, \end{cases}$$

In non-perfect conductor $\mathbf{E}$ does not vanish exactly.
$\Rightarrow$ boundary conditions do not remove wave vectors with $k_z \neq n\pi/z$.

Frequency in material with dielectric constant ϵ:
(Jackson, *Classical Electrodynamics*)

$$\omega_{\mathbf{k}} = \frac{c|\mathbf{k}|}{\sqrt{\epsilon}}$$

- Dielectric constant for perfect conductor: $\epsilon \to \infty$
- In the vacuum between the plates: $\epsilon = 1$
- Inside the plates: $\epsilon \neq 1$

$\Rightarrow$ ϵ is a function of position in space.
$\Rightarrow$ When Fourier transformed, ϵ is a function of $\mathbf{k}$ parameterized by the distance z.
$\Rightarrow$

$$\omega_{\mathbf{k}}(z) = \frac{c|\mathbf{k}|}{\sqrt{\epsilon_{\mathbf{k}}(z)}}.$$

- Casimir force due to this dependence on z (dispersion force).

Part 4.

A TOY MODEL FOR CASIMIR-LIKE EFFECT

- The full quantum description is very complicated.
- To gain intuitive understanding of full quantum description, I present a simple toy model with many qualitative features analogue to Casimir effect.

(H.N., Annals of Physics 383 (2017) 181, arXiv:1702.03291)

- Electromagnetic field $\mathbf{E}(\mathbf{x})$, $\mathbf{B}(\mathbf{x}) \rightarrow$ mimic by single degree x_1
- Charged particles $\rightarrow$ mimic by single degree x_2
- Distance between the plates $\rightarrow$ mimic by the third degree z

Hamiltonian:

$$H = \left(\frac{p_1^2}{2m} + \frac{kx_1^2}{2} \right) + \left(\frac{p_2^2}{2m} + \frac{kx_2^2}{2} \right) + \frac{p_z^2}{2M} + g(z)x_1x_2$$

Force on z:

$$\boxed{F = -\frac{\partial H}{\partial z} = -g'(z)x_1x_2}$$

To decouple x_1 and x_2, introduce new canonical variables

$$x_\pm = \frac{x_1 \pm x_2}{\sqrt{2}}, \quad p_\pm = \frac{p_1 \pm p_2}{\sqrt{2}}$$

$\Rightarrow$

$$H = H_+ + H_- + \frac{p_z^2}{2M}$$

where

$$H_\pm = \frac{p_\pm^2}{2m} + \frac{k_\pm(z) x_\pm^2}{2}, \quad k_\pm(z) = k \pm g(z)$$

Force on z in new variables:

$$\boxed{F = -\frac{g'(z)\,(x_+^2 - x_-^2)}{2}}$$

To quantize the theory we make an approximation:

- treat z as a classical background

$\Rightarrow$ quantize only the effective Hamiltonian

$$H^{(\text{eff})} = H_+ + H_-$$

$\Rightarrow$ two (quantum) uncoupled harmonic oscillators

$$H_\pm = \hbar\Omega_\pm(z)\left(a_\pm^\dagger a_\pm + \frac{1}{2}\right), \quad \Omega_\pm^2(z) = \frac{k \pm g(z)}{m}$$

effective vacuum $a_\pm|\tilde{0}\rangle = 0 \Rightarrow$

$$E_{\text{vac}}^{(\text{eff})} = \langle\tilde{0}|H^{(\text{eff})}|\tilde{0}\rangle = \frac{\hbar\Omega_+(z)}{2} + \frac{\hbar\Omega_-(z)}{2}$$

$\Rightarrow$ Casimir-like force

$$\boxed{F = -\frac{\partial E_{\text{vac}}^{(\text{eff})}}{\partial z} = -\frac{\hbar\Omega'_+(z)}{2} - \frac{\hbar\Omega'_-(z)}{2} = \frac{-\hbar g'(z)}{4m\Omega_+(z)} + \frac{\hbar g'(z)}{4m\Omega_-(z)}}$$

- Not clear how is this quantum force related to the classical force?

A Lifshitz-like approach to calculate the force:

Quantum expectation of the "classical" force operator

$$\boxed{F = -\frac{g'(z)\,\langle\tilde{0}|(x_+^2 - x_-^2)|\tilde{0}\rangle}{2}}$$

Elementary property of harmonic oscillator:

$$\langle\tilde{0}|x_\pm^2|\tilde{0}\rangle = \frac{\hbar}{2m\Omega_\pm}$$

$\Rightarrow$

$$F = \frac{-\hbar g'(z)}{4m\Omega_+(z)} + \frac{\hbar g'(z)}{4m\Omega_-(z)}$$

- the same result as with the Casimir-like approach

In both approaches, the force originates from coupling function $g(z)$.

The structure of the interacting vacuum:

In the absence of coupling $g(z) \to 0$,
- different frequency

$$\omega = \frac{k}{m} \neq \Omega_{\pm}$$

- different creation/destruction operators $a_{1,2} \neq a_{\pm}$:

$$a_j = \sqrt{\frac{m\omega}{2\hbar}} x_j + \frac{i}{\sqrt{2m\hbar\omega}} p_j$$

$$a_{\pm} = \sqrt{\frac{m\Omega_{\pm}}{2\hbar}} x_{\pm} + \frac{i}{\sqrt{2m\hbar\Omega_{\pm}}} p_{\pm}$$

Related by a Bogoliubov transformation:

$$a_\pm = \sum_{j=1,2} \alpha_{j\pm} a_j + \beta_{j\pm} a_j^\dagger$$

Bogoliubov coefficients:

$$\alpha_{1\pm} = \frac{\Omega_\pm + \omega}{2\sqrt{2\Omega_\pm \omega}}, \quad \alpha_{2\pm} = \pm\alpha_{1\pm}$$

$$\beta_{1\pm} = \frac{\Omega_\pm - \omega}{2\sqrt{2\Omega_\pm \omega}}, \quad \beta_{2\pm} = \pm\beta_{1\pm}$$

Two different vacuums $|0\rangle \neq |\tilde{0}\rangle$:

$$a_j|0\rangle = 0, \quad a_\pm|\tilde{0}\rangle = 0$$

$\Rightarrow$ The average number of free quanta $N_j = a_j^\dagger a_j$ is not zero in interacting vacuum $|\tilde{0}\rangle$:

$$\boxed{\langle\tilde{0}|N_j|\tilde{0}\rangle = \beta_{j+}^2 + \beta_{j-}^2}$$

In complete basis

$$|n, n'\rangle = \frac{(a_1^\dagger)^n (a_2^\dagger)^{n'} |0\rangle}{\sqrt{n! n'!}}$$

the interacting vacuum can be expanded as

$$|\tilde{0}\rangle = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} c_{nn'} |n, n'\rangle$$

- Explicit calculation of $c_{nn'}$ is complicated due to mixing of 2 frequencies $\Omega_\pm$.
- In the real Casimir effect (as I shall explain soon) contribution from 1 frequency can be neglected.

$\Rightarrow$ Simplified Bogoliubov transformation

$$a = \alpha(a_1 + a_2) + \beta(a_1^\dagger + a_2^\dagger)$$

- Interacting vacuum defined by $a|\tilde{0}\rangle = 0 \Rightarrow$

$$|\tilde{0}\rangle = c_0 \sum_{n=0}^{\infty} \left(-\frac{\beta}{\alpha}\right)^n |n, n\rangle$$

where $c_0 = \sqrt{1 - (\beta/\alpha)^2}$.

How is this toy model related to the real Casimir effect?

- first free oscillator analogous to electromagnetic Hamiltonian

$$\frac{p_1^2/m + kx_1^2}{2} \leftrightarrow \int d^3x \, \frac{\mathbf{E}^2 + \mathbf{B}^2}{2}$$

- second free oscillator analogous to polarization field of charged matter (J.J. Hopfield, Phys. Rev. **112**, 1555 (1958))

- the interaction term analogous to interaction between charges and electromagnetic field

$$g x_1 x_2 \leftrightarrow \int d^3x \, A_\mu j^\mu$$

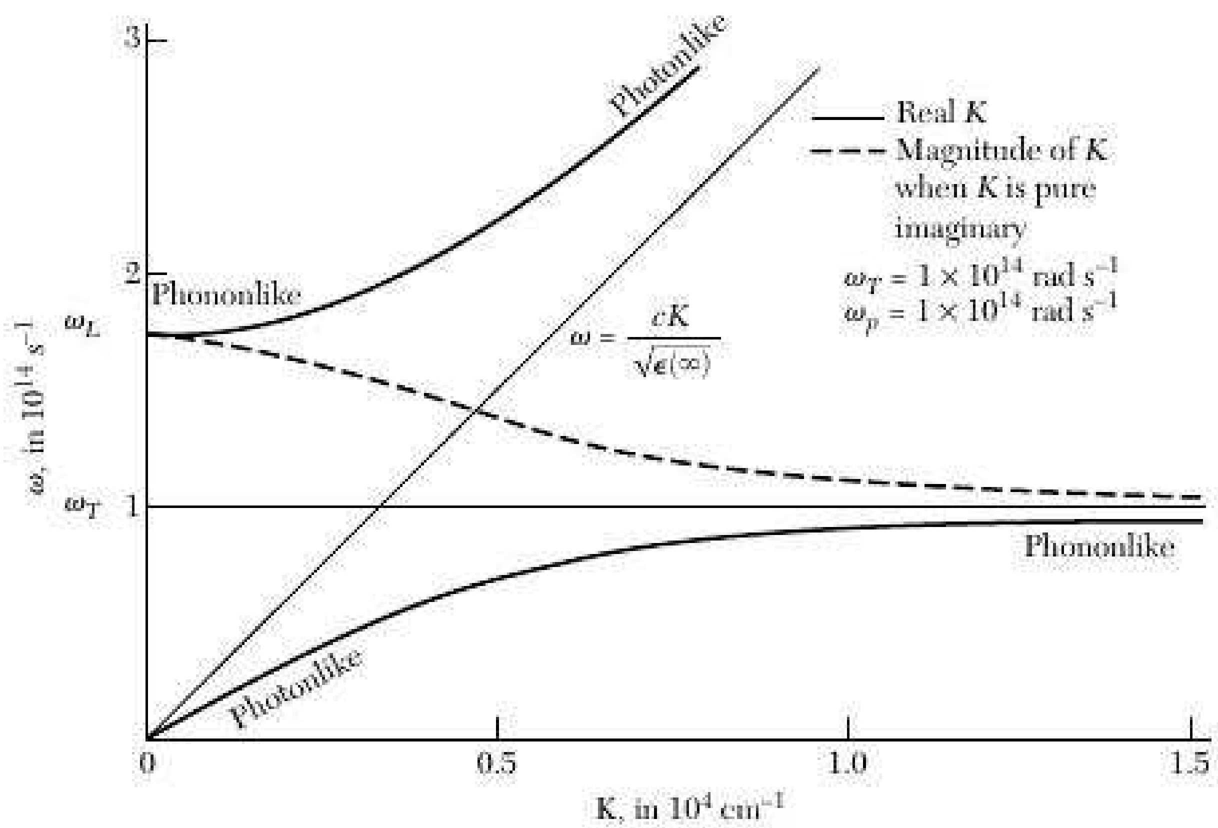
A_μ is electromagnetic 4-potential, j^μ is charged 4-current

- mixture of fundamental degrees:

$$x_+ = \frac{x_1 + x_2}{\sqrt{2}} \leftrightarrow \mathbf{D} = \mathbf{E} + \mathbf{P}$$

$\mathbf{P(x)}$ polarization (dipole moment per volume),
$\mathbf{D(x)}$ electric displacement (defined by Eq. above)

- More precisely, two frequencies $\Omega_\pm \leftrightarrow$ two branches $\omega_\pm(K)$ of the dispersion relation in a dielectric medium:



- free vacuum $|0\rangle \leftrightarrow$ state without photons and polarization quanta
- interacting vacuum $|\tilde{0}\rangle \leftrightarrow$ Casimir vacuum

- Explicit calculation of Casimir vacuum in terms of
photons and polarization quanta by Bogoliubov transformation:
F. Ciccarello *et al*, Phys. Rev. A **72**, 052106 (2005)
R. Passante *et al*, Phys. Rev. A **85**, 062109 (2012)

$\Rightarrow$ Casimir vacuum is not a state without photons
(which I also proved in Part 2).

- Casimir vacuum is a state without **polaritons**.
(W.M.R. Simpson (2015), *Surprises in Theoretical Casimir Physics*)
- Polariton is a **quasiparticle**, a complicated mixture
of photons and polarization quanta.

The final question: What is vacuum?

In physics, there are different definitions of the word “vacuum”:

1) - state without any particles

2) - state without photons

3) - state annihilated by **some** lowering operators $a_k|0\rangle = 0$

4) - local minimum of a classical potential

5) - state with lowest possible energy (ground state)

- Casimir vacuum is only 3),
it has zero number of quasiparticles (polaritons).

- Casimir vacuum is **not** 5),
for otherwise Casimir force could not attract the plates
to a state of even lower energy.